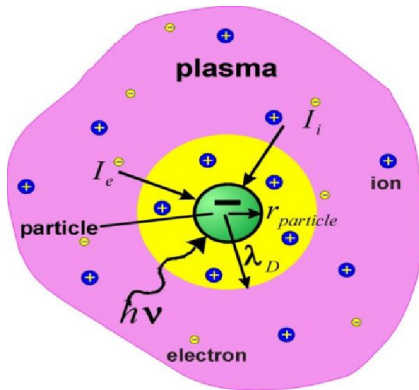


Fingerprints of different interaction mechanisms on the collective modes in complex (dusty) plasmas

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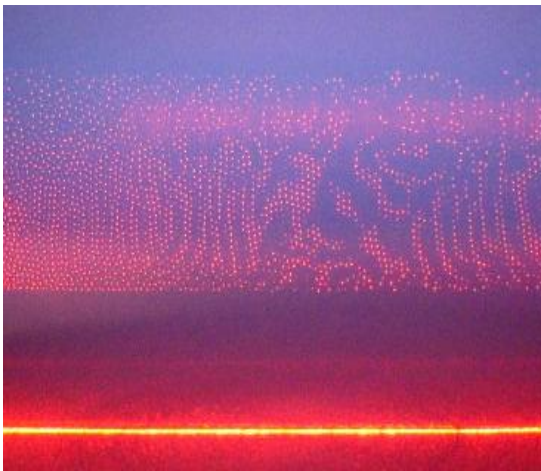
Complex Plasma: Interdisciplinary Research Field



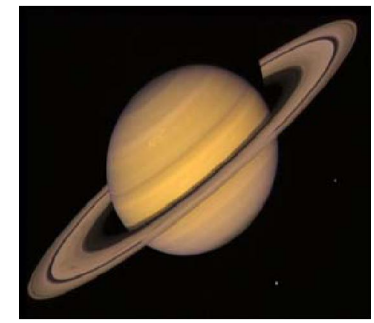
- Solid particles in the plasma background
- Particles are charged (mainly by collecting electrons and ions)
- Classical system of strongly interacting particles
- Interdisciplinary research area

Astrophysical topics:

Laboratory:



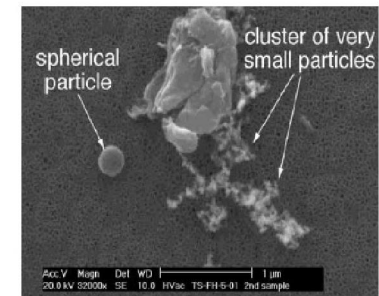
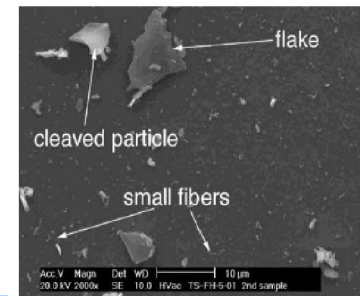
From Kretschmer, Selwyn, Sharpe, et al.



Industry:

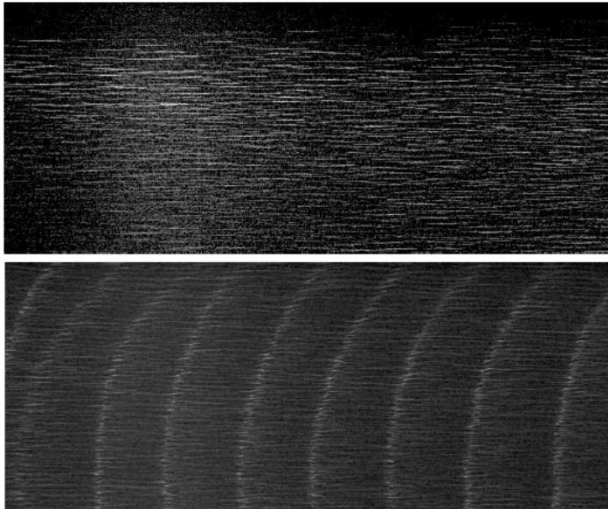


Fusion:

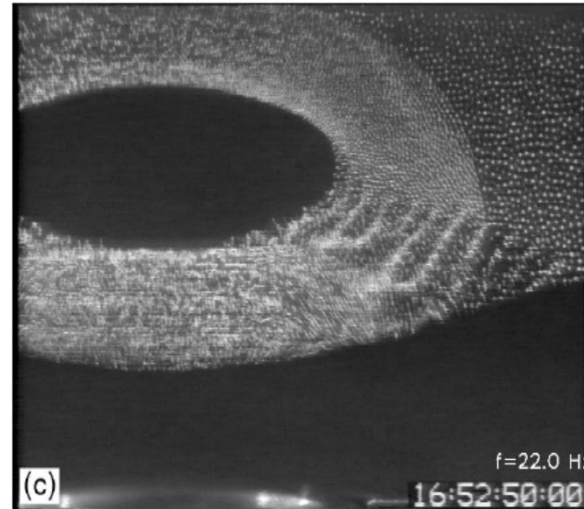


Waves in complex plasmas (few examples)

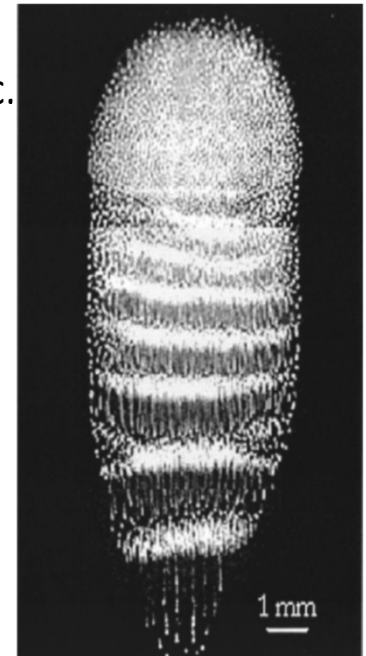
- Low damping due to particle-neutral gas collisions
- Simple determination of dispersion relation
- Individual particle motion can often be resolved
- Various topics: acoustic waves, shock waves, Mach cones, instabilities, etc.



Ratynskaia et al. (2004)



Yaroshenko et al. (2004)



Fortov et al. (2000)



Motivation:

- Central to the topic of wave phenomena in dusty (complex) plasmas is the dust acoustic velocity (DAV)
- Is DAV a universal quantity in complex (dusty) plasmas?
- What determines the magnitude of DAV when the interparticle interactions are of Yukawa type?
- What happens when interactions differ from the Yukawa type?



Theory of dust acoustic waves (DAW)

- Continuity and momentum equations for the particle component

$$\frac{\partial n_d}{\partial t} + \nabla(n_d \mathbf{v}_d) = 0 ,$$

$$\frac{\partial \mathbf{v}_d}{\partial t} + (\mathbf{v}_d \cdot \nabla) \mathbf{v}_d = -\frac{eZ}{m_d} \nabla \phi - \frac{\nabla(n_d T_d)}{m_d n_d} - \sum_{\beta} v_{d\beta} (\mathbf{v}_d - \mathbf{v}_{\beta}) .$$

- Boltzmann distribution for ions and electrons

$$n_{e(i)} \simeq n_0 \exp(\pm e \phi / T_{i(e)}) .$$

- Poisson equation

$$\nabla^2 \phi = -4\pi e (n_i - n_e + Z n_d)$$

- Standard linearization procedure [perturbations are proportional $\sim \exp(-i\omega t + i\mathbf{k}\mathbf{r})$]

$$\frac{\omega^2}{k^2} = \gamma_d v_{Td}^2 + \frac{\omega_{pd}^2 \lambda_D^2}{1 + \lambda_D^2 k^2}$$

Rao, Shukla, Yu (1990)



Dust acoustic velocity

- In the long-wavelength limit DAW exhibits acoustic dispersion with the DA velocity

$$C_{\text{DA}} = \omega_{\text{pd}} \lambda_{\text{D}} \equiv \sqrt{|Z| \frac{T_i}{T_d}} \sqrt{\frac{P\tau}{1 + (1 + P)\tau}} v_{T_d}$$

- Here $P = |Z|n_d/n_e$ is the Havnes parameter, $\tau = \frac{T_e}{T_i}$ is the electron-to-ion temperature ratio (normally ~ 100), Z is the particle charge number (normally ~ 1000)
- DA velocity can be remarkably higher than the thermal velocity (plasma related effect)



Strong-coupling effects: Fluid approach with proper thermodynamics

- The standard expression for the sound velocity of single-component fluids reads

$$c_s = v_T \sqrt{\gamma \mu}$$

where $\gamma = C_p/C_v$ is the adiabatic index, $\mu = (1/T)(\partial P/\partial n)_T$ is the isothermal compressibility modulus, and v_T is the particle thermal velocity

- Proper equation of state is required.
- The simple practical equations of state for 3D and 2D Yukawa fluids and crystals have been recently worked out in papers by Khrapak and Thomas, PRE (2015); Khrapak, Kryuchkov, Yurchenko and Thomas, JCP (2015)
- DA sound velocity can be considerably modified due to strong coupling



Dependence on coupling and screening

- Weak-coupling => conventional DAV scale

$$c_0 = \omega_p \lambda_D.$$

- Weak dependence of c_s/c_0 on Γ deep in the fluid regime
- c_s/c_0 is sensitive to the screening parameter
- c_s/c_0 drops by almost one order on the way from near-OCP to $\kappa = 5$.

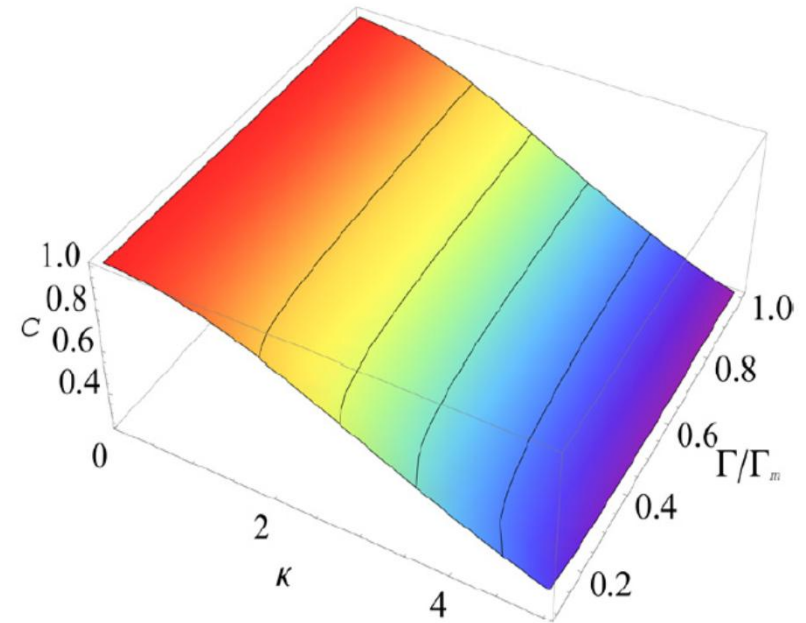


Figure 5. Three-dimensional plot of the reduced sound velocity $c \equiv c_s/c_0$ as function of Yukawa system state variables κ and Γ/Γ_m .

Khrapak and Thomas (2015)



Strongly coupled effects: Quasi-localized charge approximation (QLCA)

- Generic expressions for the longitudinal dispersion relations:

$$\omega_L^2 = \frac{n}{m} \int \frac{\partial^2 V(r)}{\partial z^2} g(r) [1 - \cos(kz)] d\mathbf{r}$$

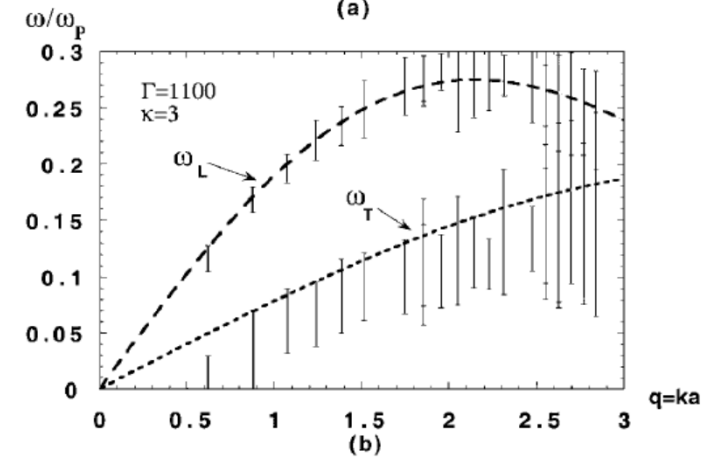
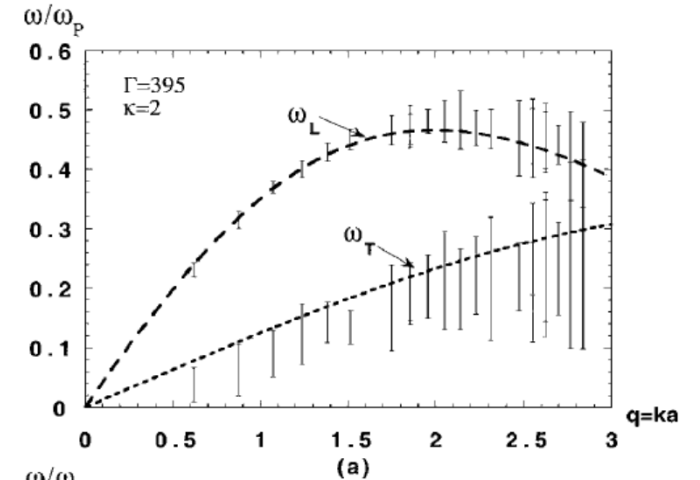
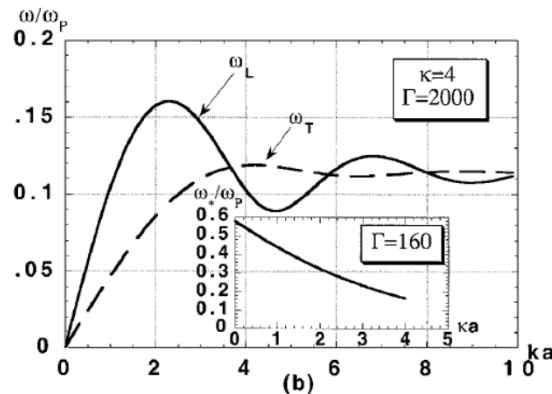
was obtained in models of collective motion in liquids by Zwanzig (1967), quasi-crystalline approximation by Hubbard&Beeby (1969), Takeno&Goda (1971). Similar expressions occur from the analysis of frequency moments of $S(k, \omega)$.

- In the context of plasma physics it is known as QLCA after Kalman and Golden applied the approximation to one-component-plasma and related systems
- For non-correlated particles [$g(r)=1$] with Yukawa interactions the integration can be performed analytically and the result is the **conventional DAW dispersion relation**



Dispersion relations at strong coupling

- **Yukawa interaction potential**
- First applied by Rosenberg and Kalman (1997) in the regime of long-wavelengths and weak screening
- Kalman et al. (2000) computed $g(r)$ using the HNC scheme get results in good agreement with MD modeling by Ohta and Hamaguchi (2000)



Kalman et al. (2000)

Relations between the fluid (thermodynamic) and QLCA approaches

- The sound velocities evaluated using the fluid thermodynamic approach and QLCA are very close, QLCA yields systematically slightly higher values
- The difference is because QLCA is a theory for high-frequency perturbations

TABLE I. Reduced sound velocity $c_s/\omega_p a$ of Yukawa fluids as calculated from the QLC approximation and present fluid model for several phase state points. QLCA data are from Ref. [29]. For details see the text.

κ	Γ/Γ_m	QLCA	Fluid
1.0	0.12	0.96	0.95
1.0	0.70	0.96	0.94
2.0	0.12	0.42	0.41
2.0	0.70	0.41	0.39
3.0	0.12	0.23	0.21
3.0	0.70	0.21	0.19

QLCA results by Kalman et al. (2000)
and Donko et al. (2008)

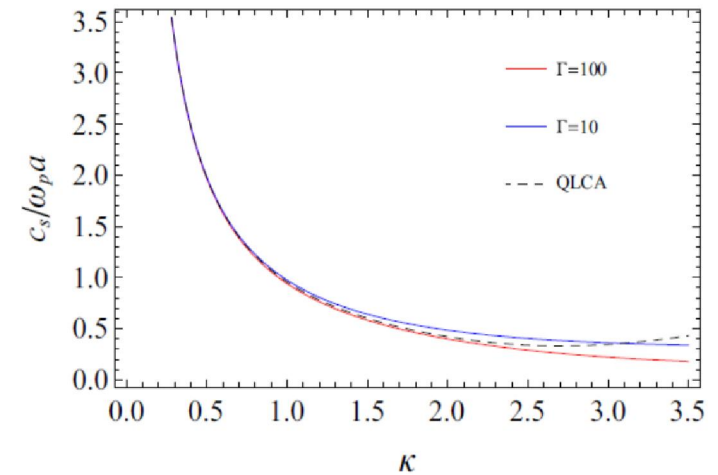


FIG. 2. (Color online) Reduced sound velocity of Yukawa fluids, $c_s/\omega_p a$, as a function of the screening parameter κ . The solid curves correspond to the results of the simple fluid approach of this paper for $\Gamma = 10$ (blue curve) and $\Gamma = 100$ (red curve). The dashed curve is plotted using QLCA result of Ref. [28], given by Eqs. (21) and (22).

Khrapak and Thomas (2015)



Actual interactions in complex plasmas

- **Can deviate from single Yukawa shape:**
 - Electron and ion collection → Power-law long-range asymptotes
 - Non-linear ion-particle interaction → Variability of the effective screening length
 - Plasma production and loss → Double-Yukawa interaction potential
 - Ion flows → Wake-mediated interaction
- **Can QLCA be used to discriminate between different interactions in complex plasmas?**



Representative examples of interaction

- Double-Yukawa potentials

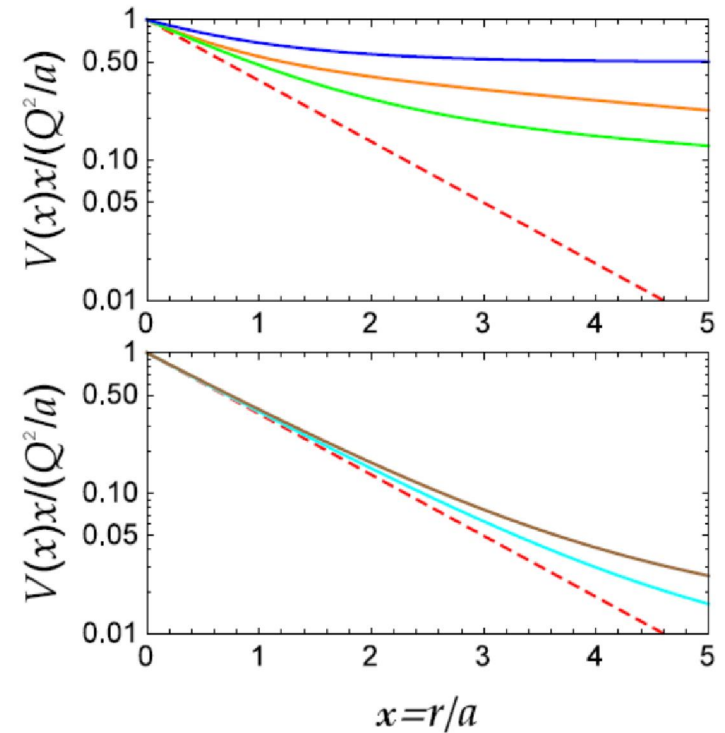
$$V(r) = \frac{Q^2}{r} [\epsilon_1 \exp(-r/\lambda_1) + \epsilon_2 \exp(-r/\lambda_2)]$$

- Yukawa + inverse square r

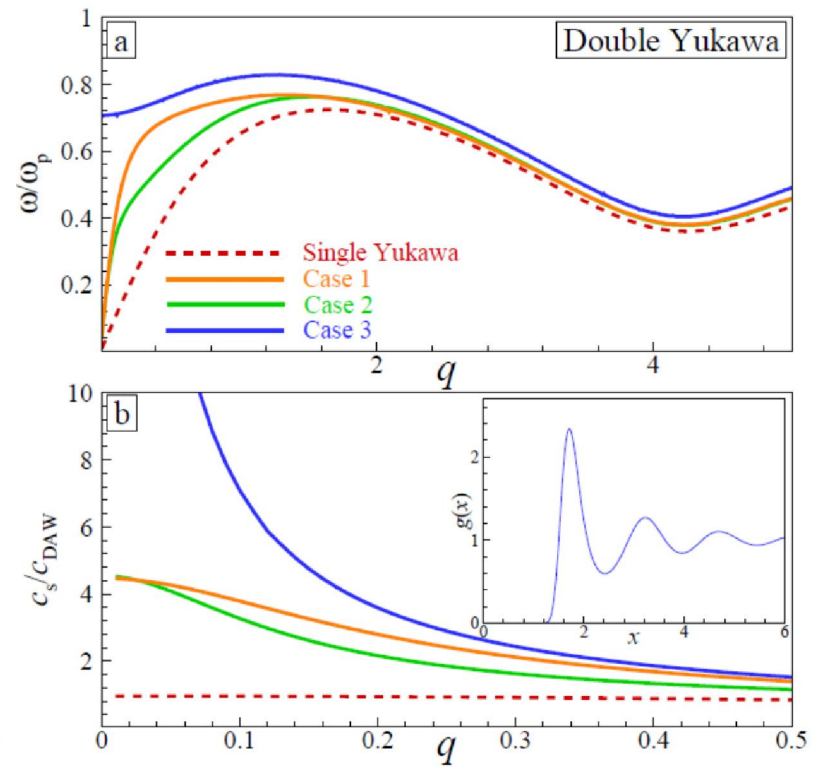
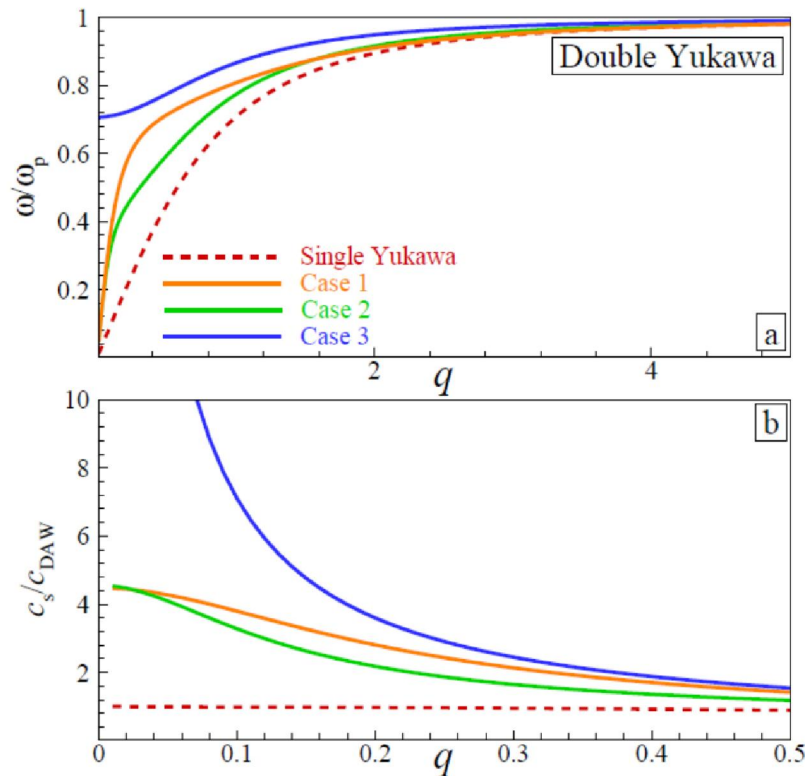
$$V(r) = \frac{Q^2}{r} \left[(1 - \epsilon) e^{-r/\lambda_D} + (\epsilon \lambda_D / r) (1 - e^{-r/\lambda_D}) \right]$$

TABLE I. Summary of the model interaction potentials considered in this study (Cases 1 - 5).

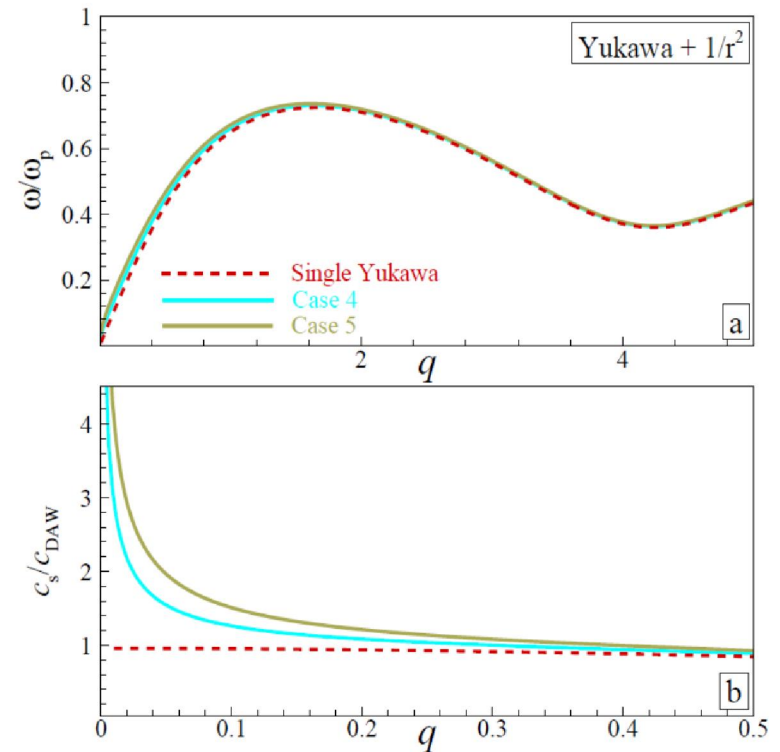
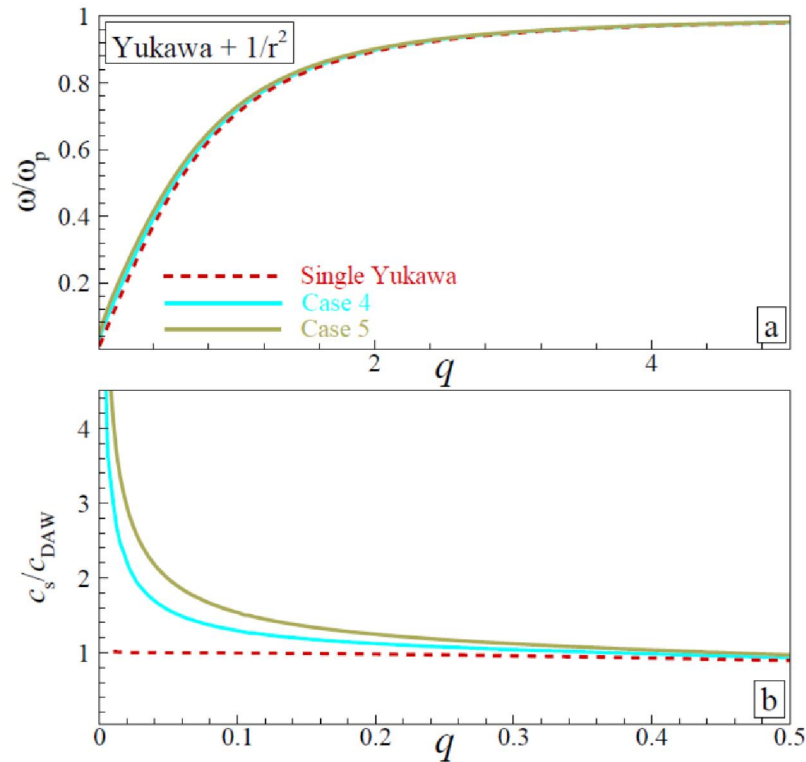
Case	Functional form	Parameters
1	Eq. (2)	$\epsilon_1 = \epsilon_2 = 0.5$, $\lambda_1 = 0.7\lambda_D$, $\lambda_2 = 6.3\lambda_D$
2	Eq. (2)	$\epsilon_1 = 0.8$, $\epsilon_2 = 0.2$, $\lambda_1 = \lambda_D$, $\lambda_2 = 10\lambda_D$
3	Eq. (2)	$\epsilon_1 = \epsilon_2 = 0.5$, $\lambda_1 = \lambda_D$, $\lambda_2 = \infty$
4	Eq. (3)	$\epsilon = 0.05$
5	Eq. (3)	$\epsilon = 0.1$



Double Yukawa class



Yukawa + inverse square class



Conclusion

- Dust acoustic velocity is NOT a universal quantity in complex (dusty) plasmas
- If the Yukawa potential can be regarded as a reasonable approximation for actual interactions, then DAV depends strongly on the screening parameter
- Deviations from Yukawa interactions have pronounced effect on the dispersion relations, especially in the long wavelength regime
- New possibility to experimentally discriminate between different interactions predicted theoretically





Thank you for your attention!

